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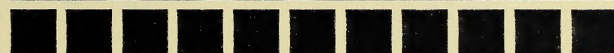
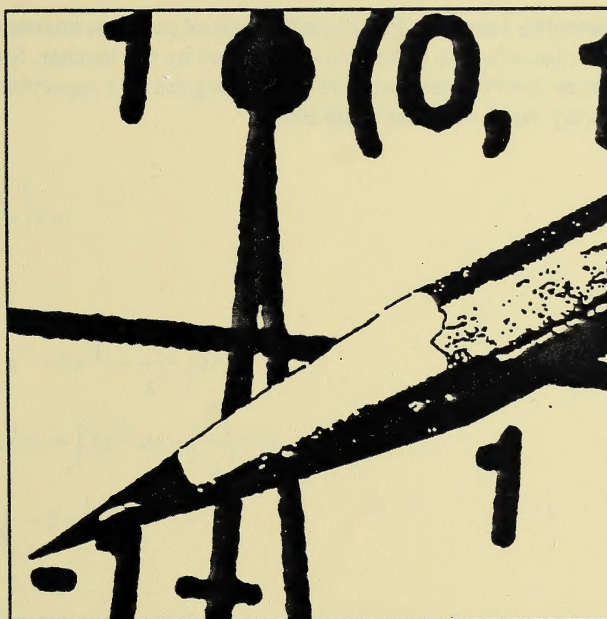
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MATHEMATICS 31

LEARNING FACILITATOR'S MANUAL



UNIT 4: INTEGRATION



**Distance
Learning**

Alberta
EDUCATION

Note

This Mathematics Learning Facilitator's Manual contains answers to teacher-assessed assignments and the final test; therefore, it should be kept secure by the teacher. Students should not have access to these assignments or the final tests until they are assigned in a supervised situation. The answers should be stored securely by the teacher at all times.

Mathematics 31
Learning Facilitator's Manual
Unit 4
Integration
Alberta Distance Learning Centre
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Topic 1: The Integral

- ⑤ 1. Determine $\int x^8 dx$.

$$\begin{aligned}\int x^8 dx &= \frac{x^{8+1}}{8+1} + c \\ &= \frac{1}{9}x^9 + c\end{aligned}$$

- ⑥ 2. Determine $\int \left(\frac{x^5 + 3x^{\frac{5}{2}} - 4}{x^2} \right) dx$.

$$\begin{aligned}\int \left(\frac{x^5 + 3x^{\frac{5}{2}} - 4}{x^2} \right) dx &= \int \left(x^3 + 3x^{\frac{1}{2}} - \frac{4}{x^2} \right) dx \\ &= \int x^3 dx + \int 3x^{\frac{1}{2}} dx - \int \frac{4}{x^2} dx \\ &= \frac{x^4}{4} + 2x^{\frac{3}{2}} + 4x^{-1} + c\end{aligned}$$

- ⑥ 3. Find the general solution of $\frac{dy}{dx} = (8x - 7)^3$.

$$y = \int (8x - 7)^3 dx$$

$$\text{Let } u = 8x - 7.$$

$$du = 8 dx$$

$$dx = \frac{1}{8} du$$

$$\begin{aligned}y &= \int \frac{u^3}{8} du \\ &= \frac{1}{8} \left(\frac{u^4}{4} \right) + c \\ &= \frac{1}{32} (8x - 7)^4 + c\end{aligned}$$

⑦

4. Solve $\frac{dy}{dx} = x(3x^2 - 2)^{\frac{1}{4}}$.

Let $u = 3x^2 - 2$.

$$du = 6x \, dx$$

$$dx = \frac{du}{6x}$$

$$y = \int x(3x^2 - 2)^{\frac{1}{4}} \, dx$$

$$= \int xu^{\frac{1}{4}} \frac{du}{6x}$$

$$= \int \frac{1}{6} u^{\frac{1}{4}} \, du$$

$$= \frac{1}{6} \left(\frac{4}{5} u^{\frac{5}{4}} \right) + c$$

$$= \frac{2}{15} (3x^2 - 2)^{\frac{5}{4}} + c$$

⑥

5. Evaluate $\int_2^8 3x^4 \, dx$.

$$\int_2^8 3x^4 \, dx = \left[\frac{3}{5} x^5 \right]_2^8$$

$$= \frac{3}{5} (8)^5 - \frac{3}{5} (2)^5$$

$$= \frac{3}{5} (32\,768 - 32)$$

$$= \frac{3}{5} (32\,736)$$

$$= 19\,641.6$$

Topic 1

_____ marks

Topic 2: The Integral and Areas

- ⑤ 1. Determine the area bounded by $y = \frac{1}{2}x^2 + 3$, the y -axis, the x -axis, and $x = 4$.

$$\begin{aligned}
 \text{Area} &= \int_0^4 \left(\frac{1}{2}x^2 + 3 \right) dx \\
 &= \left[\frac{1}{6}x^3 + 3x \right]_0^4 \\
 &= \left[\frac{1}{6}(4)^3 + 3(4) \right] - \left[\frac{1}{6}(0)^3 + 3(0) \right] \\
 &= \frac{32}{3} + 12 - 0 \\
 &= 22\frac{2}{3}
 \end{aligned}$$

- 9 2. Find the area of the regions bounded by the x -axis and the curve defined by

$$y = x^4 - x^3 - 16x^2 + 4x + 48.$$

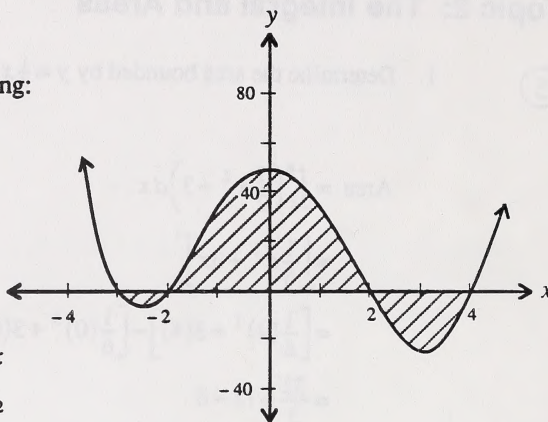
Using synthetic division, you get the following:

$$y = x^4 - x^3 - 16x^2 + 4x + 48$$

$$= (x-2)(x+2)(x+3)(x-4)$$

Therefore, the x -intercepts

are 2, -2, -3, and 4.



$$\begin{aligned} {}_{-3}A_{-2} &= \int_{-3}^{-2} (x^4 - x^3 - 16x^2 + 4x + 48) dx \\ &= \left[\frac{x^5}{5} - \frac{x^4}{4} - \frac{16x^3}{3} + 2x^2 + 48x \right]_{-3}^{-2} \\ &= \left(-\frac{32}{5} - 4 + \frac{128}{3} + 8 - 96 \right) - \left(-\frac{243}{5} - \frac{81}{4} + 144 + 18 - 144 \right) \\ &= (-6.4 - 4 + 42.67 + 8 - 96) - (-48.6 - 20.25 + 144 + 18 - 144) \\ &= -6.4 - 4 + 42.67 + 8 - 96 + 48.6 + 20.25 - 144 - 18 + 144 \\ &= -4.88\dot{3} \text{ or } -4\frac{53}{60} \end{aligned}$$

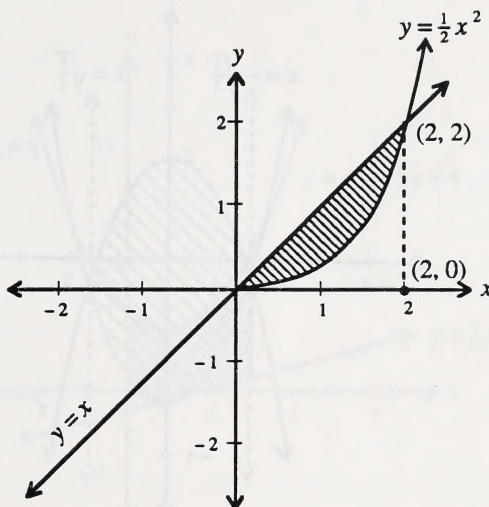
$$\begin{aligned} {}_{-2}A_2 &= \left[\frac{x^5}{5} - \frac{x^4}{4} - \frac{16x^3}{3} + 2x^2 + 48x \right]_{-2}^2 \\ &= \left(\frac{32}{5} - 4 - \frac{128}{3} + 8 + 96 \right) - \left(-\frac{32}{5} - 4 + \frac{128}{3} + 8 - 96 \right) \\ &= 6.4 - 4 - 42.67 + 8 + 96 + 6.4 + 4 - 42.67 - 8 + 96 \\ &= 119.4\dot{6} \text{ or } 119\frac{7}{15} \end{aligned}$$

$$\begin{aligned} {}_2A_4 &= \left[\frac{x^5}{5} - \frac{x^4}{4} - \frac{16x^3}{3} + 2x^2 + 48x \right]_2^4 \\ &= (204.8 - 64 - 341.33 + 32 + 192) - (6.4 - 4 - 42.67 + 8 + 96) \\ &= -40.2\dot{6} \text{ or } -40\frac{4}{15} \end{aligned}$$

$$\begin{aligned} \text{Total area} &= |-4.88\dot{3}| + |119.4\dot{6}| + |-40.2\dot{6}| \\ &= 164.61\dot{6} \text{ or } 164\frac{37}{60} \end{aligned}$$

7

3. Determine the area between the curves defined by $y = x$ and $y = \frac{1}{2}x^2$.



$$\begin{cases} y = x \\ y = \frac{1}{2}x^2 \end{cases}$$

$$\therefore x = \frac{1}{2}x^2$$

$$x^2 - 2x = 0$$

$$x(x-2) = 0$$

$$x = 0 \text{ or } x = 2$$

$$A = \int_0^2 x \, dx - \int_0^2 \frac{1}{2}x^2 \, dx$$

$$= \left[\frac{x^2}{2} \right]_0^2 - \left[\frac{x^3}{6} \right]_0^2$$

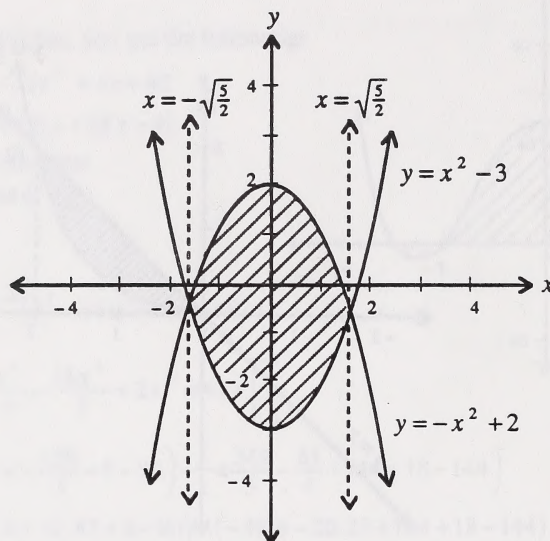
$$= \left(\frac{4}{2} - 0 \right) - \left(\frac{8}{6} - 0 \right)$$

$$= 2 - 1\frac{1}{3}$$

$$= \frac{2}{3}$$

7

4. Determine the area between the curves defined by $y = x^2 - 3$ and $y = -x^2 + 2$.



$$\begin{cases} y = x^2 - 3 \\ y = -x^2 + 2 \end{cases}$$

$$x^2 - 3 = -x^2 + 2$$

$$2x^2 = 5$$

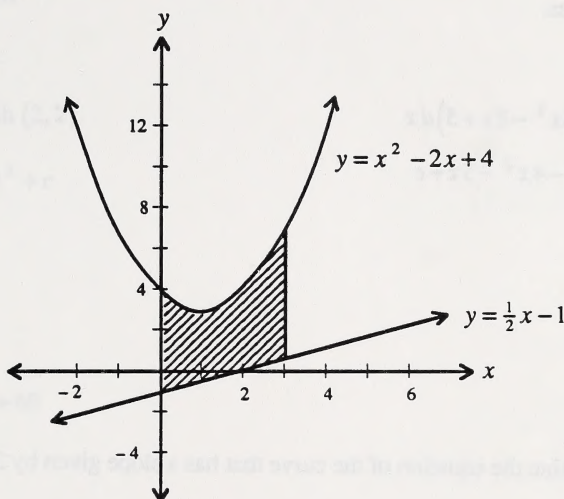
$$x^2 = \frac{5}{2}$$

$$x = \pm \sqrt{\frac{5}{2}}$$

$$\begin{aligned} A &= \int_{-\sqrt{\frac{5}{2}}}^{\sqrt{\frac{5}{2}}} (-x^2 + 2) dx - \int_{-\sqrt{\frac{5}{2}}}^{\sqrt{\frac{5}{2}}} (x^2 - 3) dx \\ &= \left[-\frac{x^3}{3} + 2x \right]_{-\sqrt{\frac{5}{2}}}^{\sqrt{\frac{5}{2}}} - \left[\frac{x^3}{3} - 3x \right]_{-\sqrt{\frac{5}{2}}}^{\sqrt{\frac{5}{2}}} \\ &= \left[-\frac{2x^3}{3} + 5x \right]_{-\sqrt{\frac{5}{2}}}^{\sqrt{\frac{5}{2}}} \\ &= \left(-\frac{5\sqrt{10}}{6} + \frac{5\sqrt{10}}{2} \right) - \left(\frac{2}{3} \cdot \frac{10\sqrt{10}}{8} - \frac{5\sqrt{10}}{2} \right) \\ &= \frac{10\sqrt{10}}{3} \end{aligned}$$

7

5. Find the area bounded by the curve $y = x^2 - 2x + 4$ and the line defined by $y = \frac{1}{2}x - 1$ over the interval $0 \leq x \leq 3$.



$$\begin{aligned}
 {}_0A_3 &= \int_0^3 \left[(x^2 - 2x + 4) - \left(\frac{1}{2}x - 1 \right) \right] dx \\
 &= \int_0^3 \left(x^2 - \frac{5}{2}x + 5 \right) dx \\
 &= \left[\frac{x^3}{3} - \frac{5}{4}x^2 + 5x \right]_0^3 \\
 &= \left(9 - \frac{45}{4} + 15 \right) - 0 \\
 &= 12.75
 \end{aligned}$$

Topic 2

_____ marks

Topic 3: Family of Curves

- ⑤ 1. The slope at any point on a curve is given by $\frac{dy}{dx} = 3x^2 - 8x + 5$. Find the equation of the family of curves.

$$\begin{aligned} y &= \int (3x^2 - 8x + 5) dx \\ &= x^3 - 4x^2 + 5x + c \end{aligned}$$

- ⑦ 2. Determine the equation of the curve that has a slope given by $3x^{\frac{1}{2}} - 4$ and passes through $(2, 5)$.

$$y = \int (3x^{\frac{1}{2}} - 4) dx$$

$$y = 2x^{\frac{3}{2}} - 4x + c$$

$$\text{At } (2, 5), 5 = 2(2)^{\frac{3}{2}} - 4(2) + c.$$

$$\therefore 5 = 4\sqrt{2} - 8 + c$$

$$c = 13 - 4\sqrt{2}$$

$$\doteq 7.34$$

$$\therefore y \doteq 2x^{\frac{3}{2}} - 4x + 7.34$$

7

3. Find the particular equation for the distance (s) if $v = 3t - 24t^2$ and $s = 2$ when $t = 2$.

$$s = \int (3t - 24t^2) dt$$

$$s = \frac{3}{2}t^2 - 8t^3 + c$$

Replace (s, t) with $(2, 2)$.

$$2 = \frac{3}{2}(2)^2 - 8(2)^3 + c$$

$$2 = 6 - 64 + c$$

$$c = 58 + 2$$

$$= 60$$

$$\therefore s = \frac{3}{2}t^2 - 8t^3 + 60$$

8

4. The velocity of a moving object is given by $v = 6t^2 - 4t + 12$. Determine the distance the object moves in metres from $t = 4$ s to $t = 7$ s.

$$s = \int (6t^2 - 4t + 12) dt$$

$$= 2t^3 - 2t^2 + 12t + c$$

$$\text{At } t = 7, s_1 = 2(7)^3 - 2(7)^2 + 12(7) + c.$$

$$\therefore s_1 = 686 - 98 + 84 + c$$

$$= 672 + c$$

$$\text{At } t = 4, s_2 = 2(4)^3 - 2(4)^2 + 12(4) + c.$$

$$\therefore s_2 = 128 - 32 + 48 + c$$

$$= 144 + c$$

$$s_1 - s_2 = 672 - 144$$

$$= 528$$

Therefore, the object moves 528 m.

8

5. A rock is dropped from a building. The acceleration due to gravity is -9.8 m/s^2 . Find the distance travelled by this rock from $t = 2 \text{ s}$ to $t = 4 \text{ s}$.

$$a = -9.8 \text{ m/s}^2$$

$$v = \int -9.8 \, dt$$

$$= -9.8t + c$$

$$\text{When } t = 0, v = 0.$$

$$\therefore 0 = -9.8(0) + c$$

$$c = 0$$

$$v = -9.8t$$

$$s = \int -9.8t \, dt$$

$$= \frac{-9.8t^2}{2} + k$$

$$= -4.9t^2 + k$$

$$\text{When } t = 0, s = 0; \text{ therefore, } k = 0.$$

$$s = -4.9t^2$$

$$\text{At } t = 4, s = -4.9(4)^2.$$

$$\therefore s = -78.4$$

$$\text{At } t = 2, s = -4.9(2)^2.$$

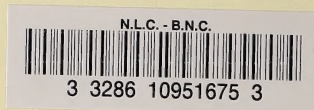
$$\therefore s = -19.6$$

$$\begin{aligned} \text{Distance travelled} &= -78.4 - (-19.6) \\ &= -58.8 \end{aligned}$$

The rock travels 58.8 m.

Topic 3

_____ marks



This booklet cannot be purchased separately; the
Learning Facilitator's Manual is available
only as a complete set.

